Magnetic traps and evaporative cooling

Note Title 2/2/2005

Let's say that you want to trap some atoms — you could use an electric trap for ions and a magnetic trap for neutrals

ions — $U = q\Phi$ electric potential
neutrals — $U = -\vec{\mu}\cdot\vec{B}$ magnetic dipole interaction

What are your choices if you limit yourself to static (or quasi-static fields)?

Electric trap: Earnshaw's theorem d'oh!

- no stable equilibrium for a charge in a static potential

$\nabla^2\Phi = 0$ $\quad \dfrac{\partial^2\Phi}{\partial x^2} + \dfrac{\partial^2\Phi}{\partial y^2} + \dfrac{\partial^2\Phi}{\partial z^2} = 0$

- so, curvature must have different signs in different directions — that's not confining!

Solution: rf Paul trap, Penning trap - more later

Magnetic trap: Wing's theorem d'oh!

- There can be no local maxima in a source-free region for static/quasi-static magnetic, electric, or gravitational fields

Now there are absolute maxima (at the sources, like currents) which means that "strong field seekers" cannot be trapped (they will just run into the sources)

$\mu < 0$ - weak-field seeker for magnetic trap - only trappable states

Proof of Wing's theorem (Wing, Progress in Quantum Electronics, 8, 181 (1984))

Two parts: ① no local maxima by contradiction
② local minimum by construction

① suppose a field $\vec{F}$ has a local maximum in a source-free region at $\vec{r}=0$

then. $\vec{F}(\vec{r}) = \vec{F}(0) + \delta F(\vec{r})$

$$F(r)^2 = F(0)^2 + \delta F(r)^2 + 2\vec{F}(0)\cdot\delta\vec{F}(\vec{r})$$

→ if $F(0)^2 > F(r)^2$, then $\vec{F}(0)\cdot\delta\vec{F}(\vec{r}) < 0$

since $\delta F(r)^2 > 0$

→ take $\vec{F}$ along $\hat{z}$, then

$$F_z(0)\,\delta F_z(r) < 0$$

$$\delta F_z(r) < 0 \quad \text{I}$$

in a source-free region, $\vec{\nabla}\cdot\vec{F}=0$

$\vec{\nabla}\times\vec{F}=0$

using: $\vec{\nabla}(\vec{\nabla}\cdot\vec{F}) - \vec{\nabla}\times(\vec{\nabla}\times\vec{F}) = \nabla^2 F \rightarrow \nabla^2\vec{F} = 0 \rightarrow \left.\begin{array}{l}\nabla^2 F_x = 0\\ \nabla^2 F_y = 0\\ \nabla^2 F_z = 0\end{array}\right\}$

Now, $\delta\vec{F}(\vec{r})$ must also be a solution to the field equations, so $\nabla^2\delta\vec{F} = \nabla^2\delta F_z = 0$

Now bust out Green's theorem:

for any scalar-fields ψ and ϕ:

$$\int_V \left(\psi \nabla^2\phi - \phi\nabla^2\psi\right) d\tau = \int_S \left(\psi\vec{\nabla}\phi - \phi\vec{\nabla}\psi\right)\cdot d\vec{a}$$

we choose: $\begin{cases} \psi = \delta F_z(r) \\ \phi = \frac{1}{r} \longrightarrow \nabla^2\phi = 4\pi\delta(r) \end{cases}$

$$\vec{\nabla}\phi = -\frac{3}{r^2}\hat{r}$$

- and the volume V is a sphere of radius r centered at $\vec{r}=0$

the l.h.s. of Green's theorem is: $\int_V \left(4\pi\delta F_z\delta(r) - \frac{1}{r}\cancel{\nabla^2\delta F_z}^{\,0}\right)$

$$= \delta F_z(0) = 0$$

the r.h.s. is: $\int_S \left(-\delta F_z \frac{3}{r^2}\hat{r} - \frac{1}{r}\vec{\nabla}\delta F_z\right)\cdot d\vec{a}$

now: $\int_S \left(\frac{1}{r}\vec{\nabla}\delta F_z\right)\cdot da = \frac{1}{r}\int_V \vec{\nabla}\cdot(\vec{\nabla}F)\, d\tau =$

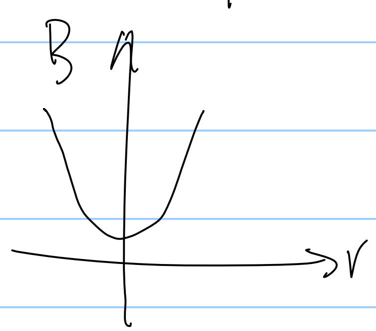
$$\frac{1}{r}\int_V \nabla^2\delta F\, d\tau = 0$$

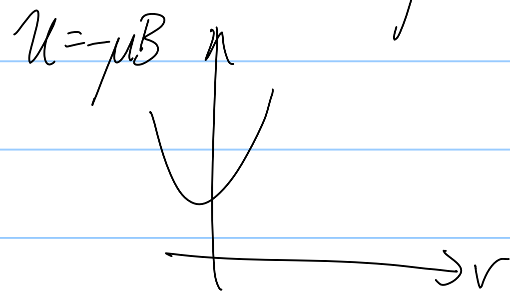
by Gauss' theorem

so, the r.h.s. $= -\frac{3}{r^2}\int_S \delta F_z \, da$, which is an average of δF_z over the sphere — this can only be 0 if δF_z is not always <0, so we have a contradiction with I

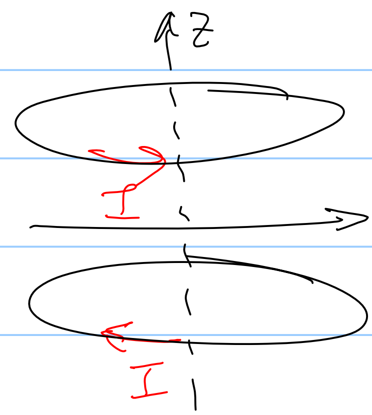
Magnetic traps

- Always trap around a minimum in B
- only atomic states with $\mu < 0$ are trapped





1st trap: quadrupole trap



near the minimum, $\vec{B} = B' x \hat{x} + B' y \hat{y} - 2B' z \hat{z}$

$$B = B' \sqrt{x^2 + y^2 + 4z^2}$$

gradient B' typically $\gtrsim 300$ G/cm

→ the field direction changes in space, and, to stay trapped, the atom's magnetic moment must adiabatically follows

Adiabatic theorem in quantum mechanics

Hamiltonian changes gradually: $H_i \to H_f$

adiabatic theorem: if quantum state was the n^{th} eigenstate of H_i, ends up in the n^{th} eigenstate of H_f (assuming no degeneracy, which we can relax)

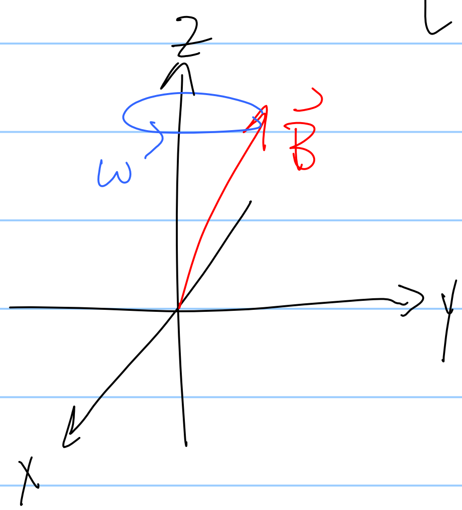
proof — see Griffiths, Chapter 10

$$H = H_0 + H'(t) \qquad H'(t) = V f(t)$$

require: $\frac{df}{dt} \ll \frac{\Delta E}{\hbar} f$ where ΔE is a characteristic energy gap

generally: adiabaticity requires H to change on timescales which are slow compared with "internal" timescales relevant example: spin-1/2 particle in a magnetic field that is changing direction in time

$$B(t) = B_0 \left[\sin\alpha \cos(\omega t)\,\hat{x} + \sin\alpha \sin(\omega t)\,\hat{y} + \cos\alpha\,\hat{z} \right]$$



$$H = -\vec{\mu}\cdot\vec{B} = 2\frac{\mu_0}{\hbar}\vec{S}\cdot\vec{B} = \mu_0 B_0 \left\{ \sin\alpha\left[\cos(\omega t)\sigma_x + \sin(\omega t)\sigma_y\right] + \cos\alpha\,\sigma_z \right\}$$

$$= \frac{\hbar\omega_e}{2}\begin{pmatrix} \cos\alpha & e^{-i\omega t}\sin\alpha \\ e^{i\omega t}\sin\alpha & -\cos\alpha \end{pmatrix}$$

$\omega_e = \frac{2\mu_0 B}{\hbar}$ Larmor frequency eigenvectors of H: $\chi_+ = \begin{pmatrix} \cos \alpha/2 \\ e^{i\omega t} \sin \alpha/2 \end{pmatrix}$ $\quad \chi_- = \begin{pmatrix} \sin \alpha/2 \\ -e^{i\omega t} \cos \alpha/2 \end{pmatrix}$

spin up and down along the instantaneous direction of B

$$E = \pm \frac{\hbar \omega_e}{2}$$

if $\chi(t=0) = \begin{pmatrix} \cos \alpha/2 \\ \sin \alpha/2 \end{pmatrix}$ $\quad$ spin up along $\vec{B}(t=0)$

Exact solution (just use Schrödinger's equation):

$$\chi(t) = \begin{pmatrix} \left[\cos\left(\frac{\lambda t}{2}\right) + i \frac{\omega - \omega_e}{\lambda} \sin\left(\frac{\lambda t}{2}\right)\right] \cos \alpha/2 \, e^{-i\omega t/2} \\ \left[\cos\left(\lambda t/2\right) - i \frac{\omega + \omega_e}{\lambda} \sin\left(\frac{\lambda t}{2}\right)\right] \sin \alpha/2 \, e^{i\omega t/2} \end{pmatrix}$$

where $\lambda = \sqrt{\omega^2 + \omega_e^2 - 2\omega\omega_e \cos\alpha}$

rewriting in terms of eigenspinors:

$$\chi(t) = \left[\cos\frac{\lambda t}{2} + i\,\frac{\omega\cos\alpha - \omega_e}{\lambda}\sin\frac{\lambda t}{2}\right]e^{-i\omega t/2}\chi_+(t) +$$

$$i\left[\frac{\omega}{\lambda}\sin\alpha\,\sin\frac{\lambda t}{2}\right]e^{+i\omega t/2}\chi_-(t)$$

$$|\langle\chi(t)|\chi_-(t)\rangle|^2 = \left[\frac{\omega}{\lambda}\sin\alpha\,\sin\frac{\lambda t}{2}\right]^2$$ exact transition probability

if $\omega \ll \omega_e$ — we should be adiabatic!

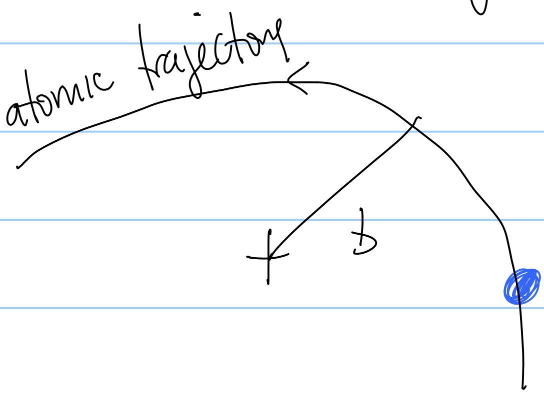
$\omega \ll \omega_e$ means $\lambda \to \omega_e$

$$|\langle\chi(t)|\chi_-(t)\rangle|^2 \to \left(\frac{\omega}{\omega_e}\sin\alpha\,\sin\frac{\omega_e t}{2}\right)^2 \to 0$$

So, the spin will track the direction of the magnetic field

For a quadrupole trap, adiabaticity is violated near the center, where the Larmor frequency is small!

consider a specific trajectory:



b is minimum distance from $r=0$

if $\frac{\mu b B'}{\hbar} \ll \frac{v}{b}$, then there can be a spin-flip to an untrapped state

$\frac{\mu b B'}{\hbar}$: Larmor frequency at b

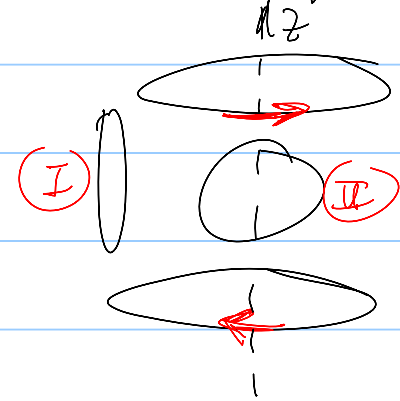
$\frac{v}{b}$: rate of change of field direction

This condition sometimes stated as: $\underbrace{\frac{\vec{v}\cdot\vec{\nabla}B}{B}}_{\text{convective derivative}} \ll \frac{\mu B}{\hbar}$

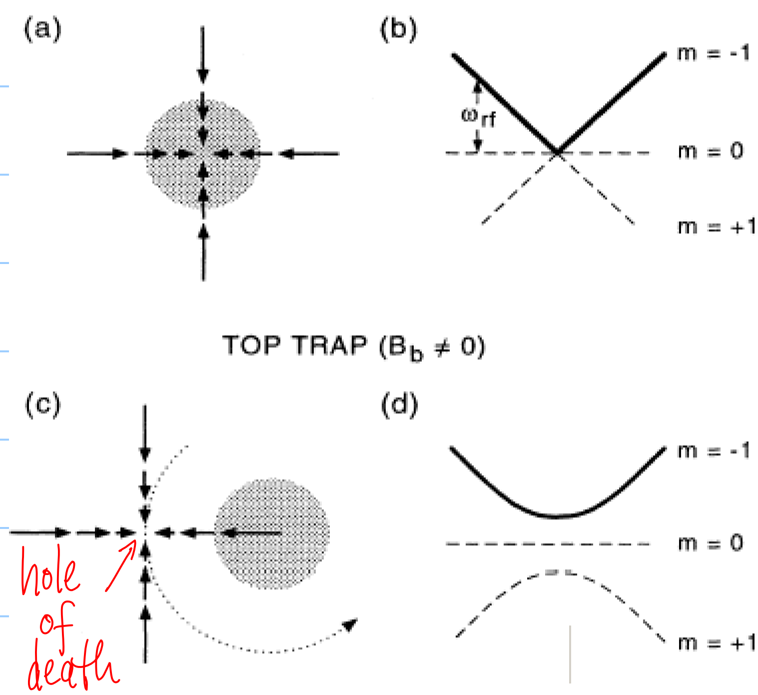
Solutions to plugging the hole: ① TOP trap (Eric Cornell)
② optical plug (Wolfgang Ketterle)
③ Ioffe-Pritchard trap

$\boxed{\text{TOP trap}}$: take a quadrupole trap and add a rotating bias field: PRL 74, 3352 (1995)

coils ⓘ and ⓘⓘ driven out-of-phase at ω_b (~10 kHz)



the time-averaged potential is harmonic:
(provided that the drive frequency >> harmonic frequency for motion)



$\boxed{\text{In a TOP trap, the direction of the atomic magnetic moment is rotating with respect to the lab frame!}}$

$$\vec{B} = \left(B'x + B_0 \cos\omega_b t\right)\hat{x} + \left(B'y + B_0 \sin\omega_b t\right)\hat{y} - 2B'z\,\hat{z}$$

$$B = \sqrt{\left(B_0 \cos\omega_b t + B'x\right)^2 + \left(B_0 \sin\omega_b t + B'y\right)^2 + 4B'^2 z^2}$$

for $r \ll \frac{B_0}{B'}$ and averaging over one period of ω_b (see Pethick and Smith, Chapter 4)

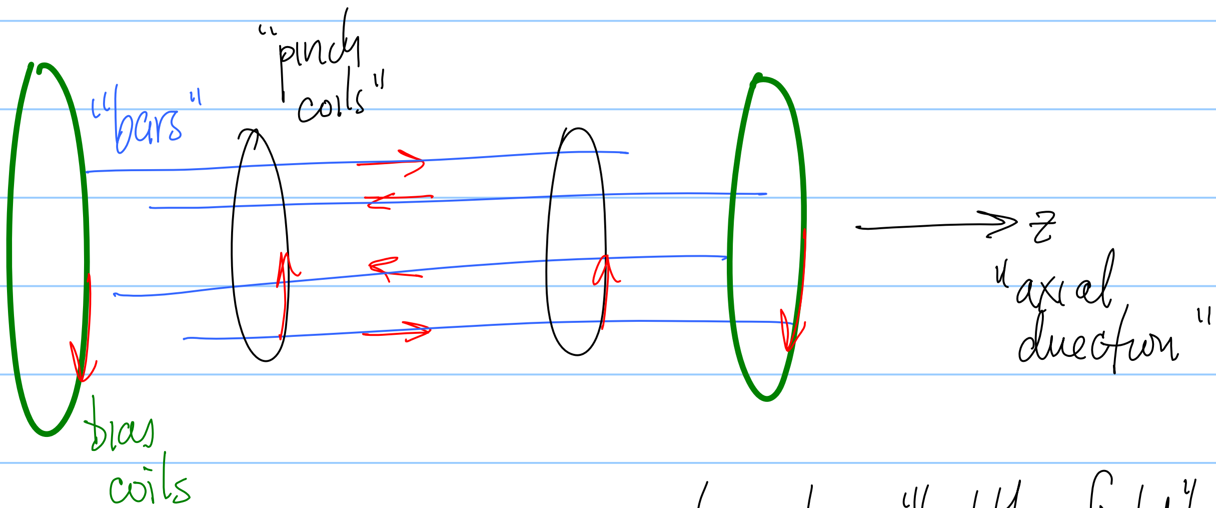
$$\boxed{\langle B \rangle_{\omega_b} \simeq B_0 + \frac{B'^2}{4B_0}\left(r^2 + 8z^2\right)}$$

- B_0 typically 10 G, leading to harmonic oscillator frequencies ~ 100 Hz

- There are tricks that one can play to make this trap very spherical (important for vortex in BEC physics - see Jason Ensher's thesis) using gravity

Ioffe-Pritchard trap

classic design:



- pinch coils – "bottle field" – confinement in z
- bars – quadrupole field in x and y.
- bias coils – subtracts from pinch coil field controls overall bias field

$$B_z = \underbrace{B_0}_{\text{bias + pinch}} + \underbrace{B_2\left(z^2 - \rho^2/2\right)}_{\text{pinch – has small contribution}}$$

$$B_\rho = -\underbrace{B_2 \rho z}_{\text{pinch}} + \underbrace{C_1 \rho \cos(2\varphi)}_{\text{bars}}$$

$$B_\varphi = -\underbrace{C_1 \rho \sin(2\varphi)}_{\text{bars}}$$

these coefficients, B_0, B_2, C_1, have to be calculated in general;
$C_1 \propto I_{bars}$,
$B_2 \propto A I_{pinch} - B I_{bias}$,
$B_0 \propto C I_{pinch} - D I_{bias}$

near the center of the trap:

$$|B| \simeq B_0 + B_2 z^2 + \frac{C_1^2 - B_0 B_2}{2B_0} \rho^2$$

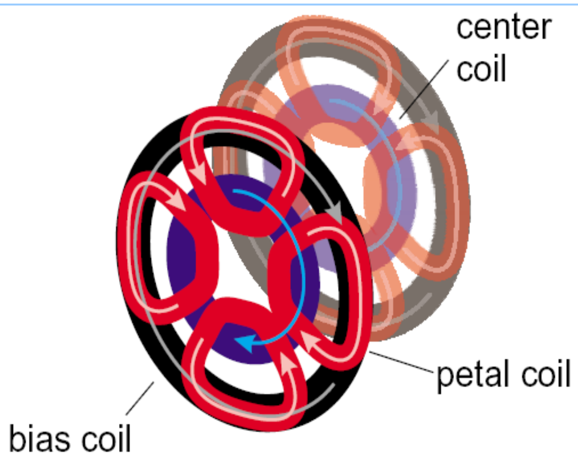
Note that radial curvature depends on the bias field B_0 ; $C_1 \sim 350$ G/cm, $B_0 \sim 1$ G, $B_2 \sim 200$ G/cm^2

Also, aspect ratio is adjustable:

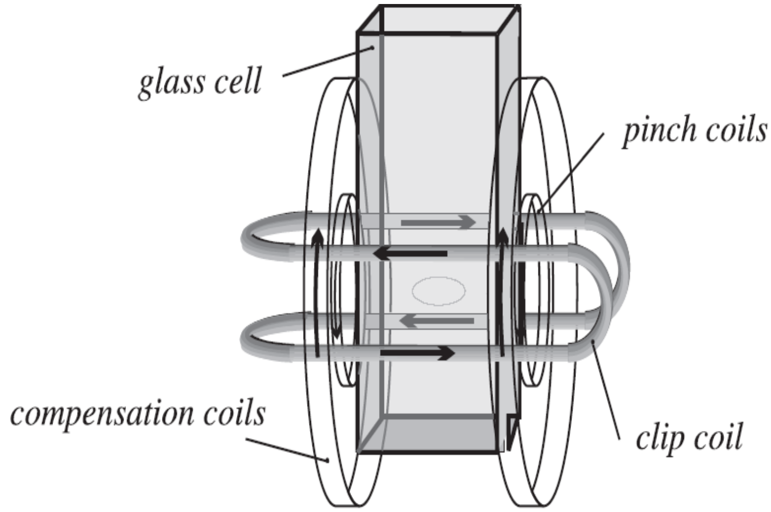
$$\omega_z = \sqrt{\frac{\mu}{m} 2B_2} \qquad \omega_r = \sqrt{\frac{\mu}{m} \frac{C_1^2 - B_0 B_2}{B_0}}$$

$\frac{\omega_r}{\omega_z}$ depends on on current through bar & pinch coils, but also $\underline{B_0}$

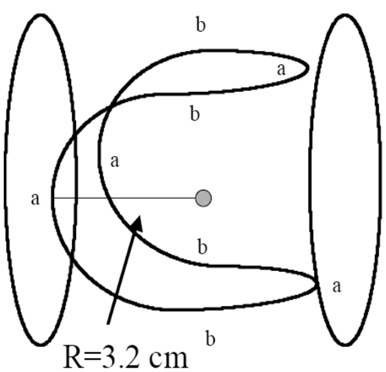
Various flavors



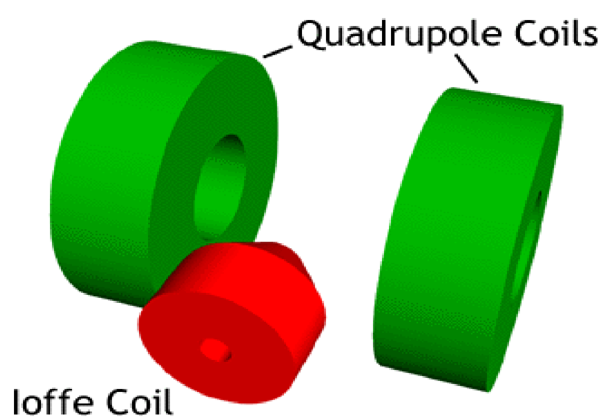
cloverleaf trap



standard I-P / "clip"



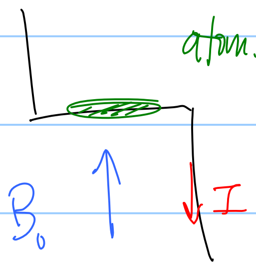
baseball



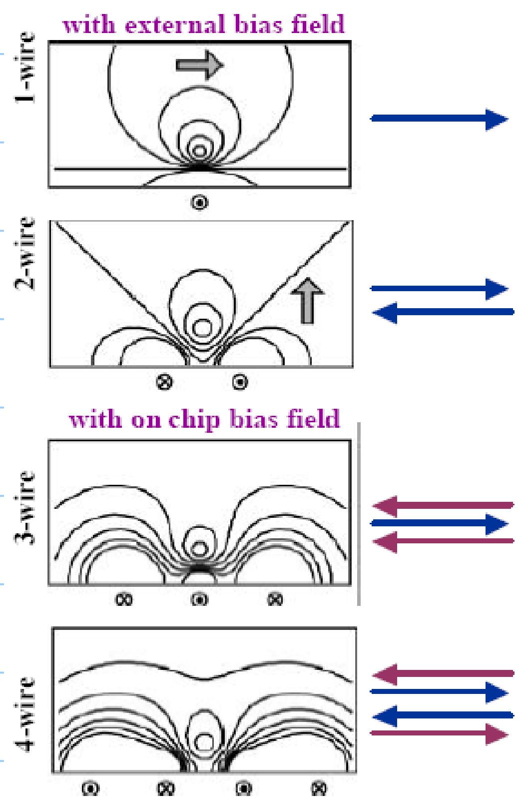
QUIC trap

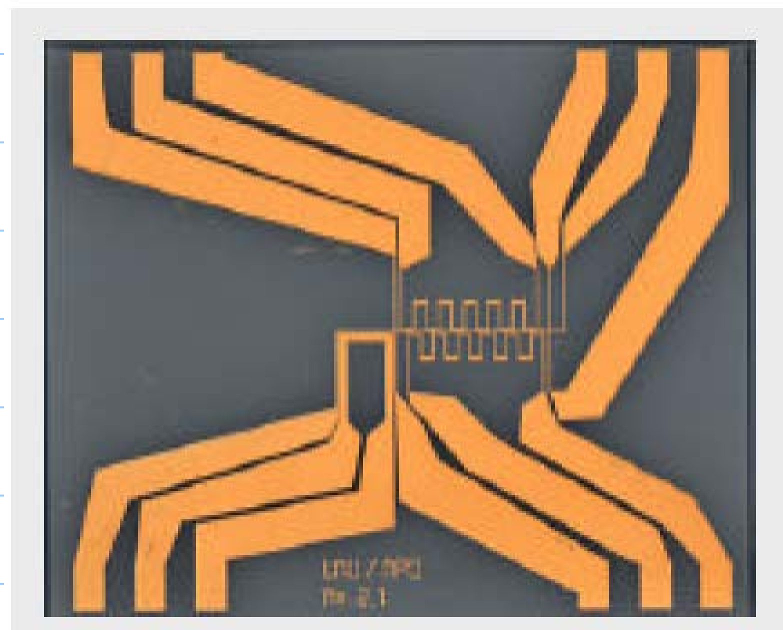
Chip trap:

simplest version:

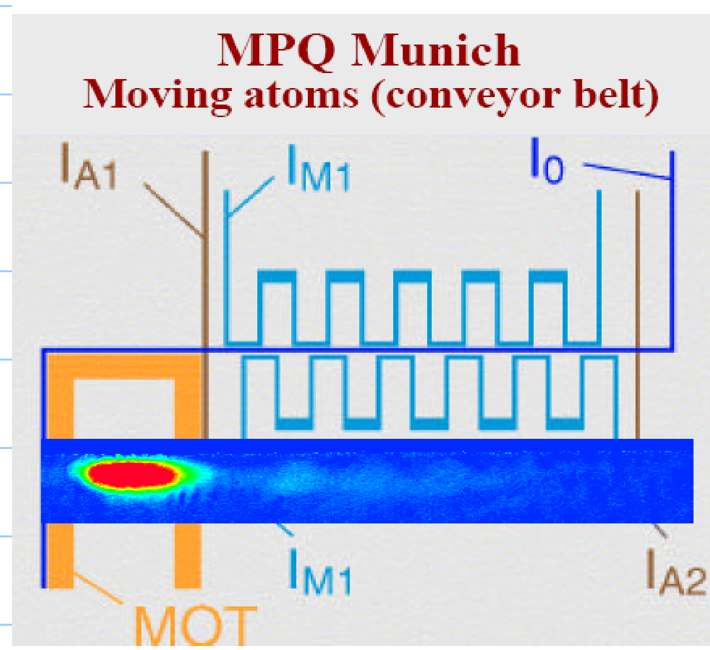


atoms trapped above the surface

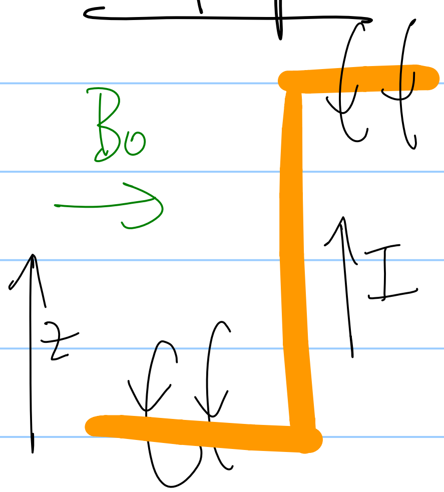


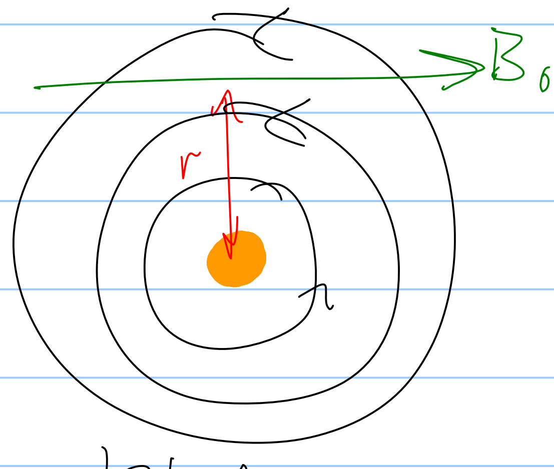


~20 mm

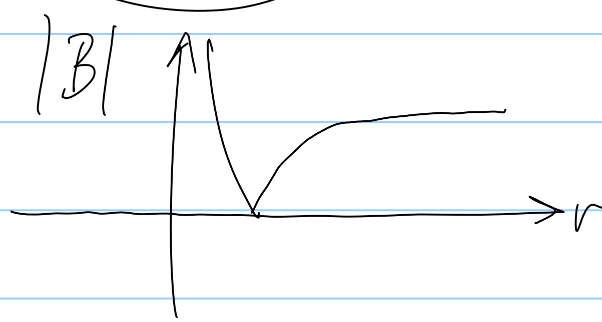


Z-trap

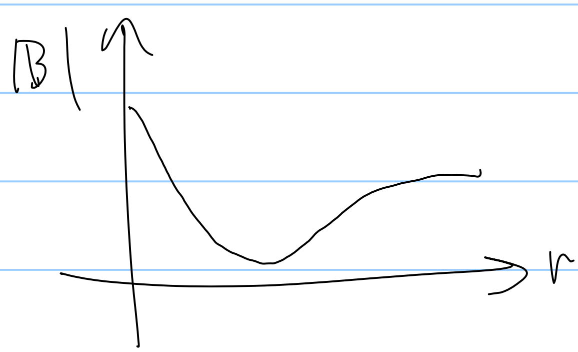


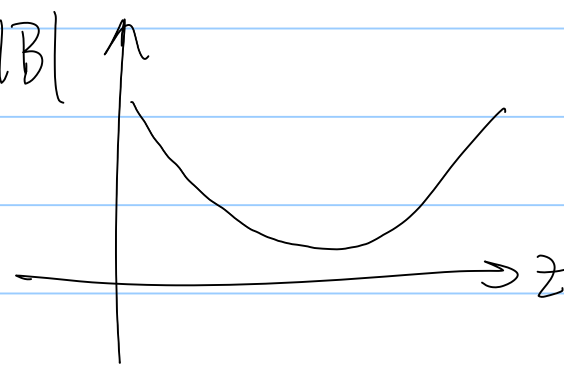


bias field cancels the field from a long straight wire at $r = \frac{\mu_0}{2\pi}\frac{I}{B_0}$



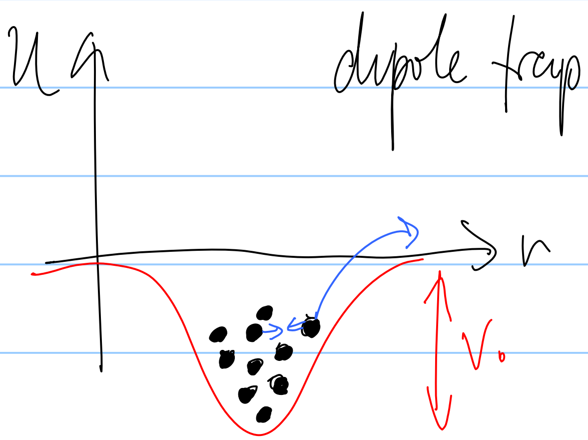
The other legs of the "Z" provide a non-zero bias field roughly parallel to the wire and confinement along z, creating a Ioffe-Pritchard style trap!

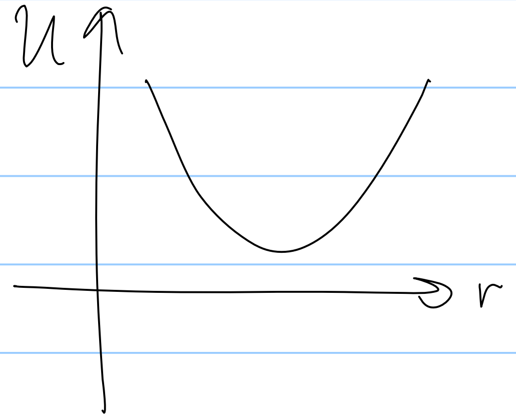




Evaporative cooling

Combined w/ conservative trapping potentials as a route to quantum degeneracy

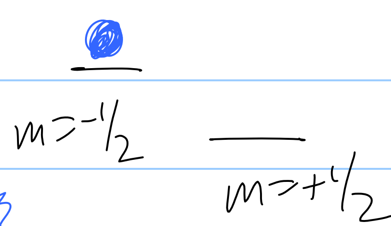




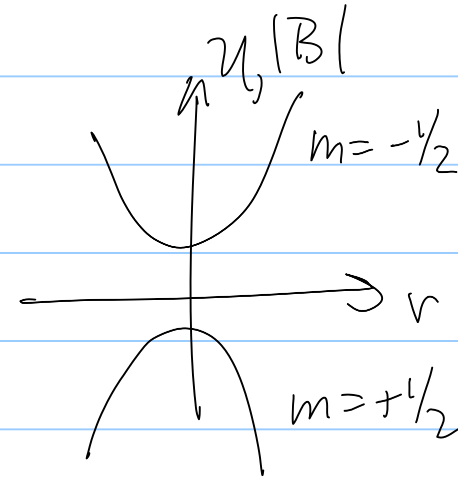
collisions promote atoms to energies greater than the trap depth, resulting in cooling (after rethermalization via more collisions)

We make a magnetic trap have finite depth using rf/microwave coupling between spin states

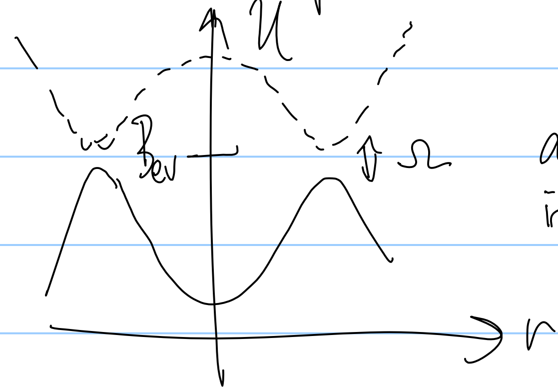
Imagine a spin $\frac{1}{2}$ atom in a magnetic field:



magnetically trappable



turn on oscillating magnetic field which is resonant with the transition at some field B_{ev} $\quad |\tfrac{1}{2}\rangle \Leftrightarrow |-\tfrac{1}{2}\rangle$



adiabatic potential — if an ~~atom~~ atom adiabatically follows the local Hamiltonian, i.e. no Landau-Zener transitions

This is a finite-depth trap, and the atoms can evaporate!

In evaporative cooling, we roughly keep η fixed, where the trap depth $= \eta k T$.

This means that we continually lower the laser intensity for a dipole trap, or continually lower the rf frequency for a magnetic trap.

How does evaporative cooling work?

Specialize to a 3D harmonic potential

Using simple thermodynamic arguments, if the average energy carried away by an escaping atom is ηkT, then, with $\alpha \approx \frac{\eta}{3} - 1$, $\frac{d(\ln T)}{d(\ln N)} = \alpha$

The goal in evaporative cooling is to get the biggest change in T with the smallest change in N... and here's why: we want to increase phase space density $\mathcal{D}$.

$$\mathcal{D} = n_{pk} \lambda_{dB}^3 \qquad \lambda_{dB} = \sqrt{\frac{h}{2\pi m k T}}$$

in a harmonic trap, $\frac{1}{2} m \omega^2 \sigma^2 = \frac{1}{2} kT$ where σ is the size of the gas

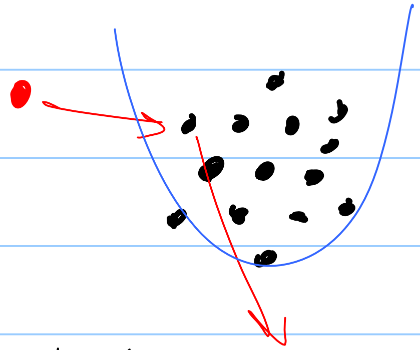
$$\sigma \propto \sqrt{T}$$

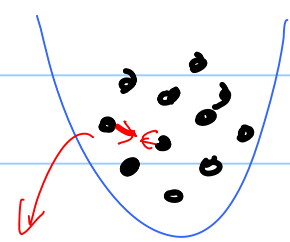
and $n_{pk} \propto \frac{N}{\sigma^3} \propto \frac{N}{T^{3/2}}$

therefore, $\mathcal{D} \propto \frac{N}{T^{3/2}} \cdot \frac{1}{T^{3/2}} \propto \frac{N}{T^3}$

The best strategy in a perfect world: make η very large, and wait (for a very long time) for one atom to carry away all of the energy.

The problem: there are competing loss processes, such as collisions with residual gas atoms and inelastic collisions.





residual gas atoms — indescriminantly remove high and low energy atoms

spin changing collisions — remove low (potential) energy collisions, since that collision rate scales like a power of n

So, what sets whether or not this will work to bring $D \to 1$ (starting from 10^{-6} or 10^{-7} in a MOT)?

$$R = \frac{R_{good}}{R_{bad}}$$

R_{good} ← elastic collision rate

R_{bad} ← "bad" collision rate

$$R = \frac{\tau_{bad}}{\tau_{el}}$$

How to show this?

first, what's the rate at which atoms leave the 'trap'?

$R_{good} = 1/\tau_{el} = n_{dwd}\, \sigma\, v_r$ ← mean relative speed $\propto \sqrt{T}$

collision cross-section

density-weighted-density $\propto n_{pk}$

you can show that $\dot{N} \simeq -N n_{pk} \sigma \bar{v} \eta e^{-\eta} = -N/\tau_{ev}$

<u>pro tip / think</u>: velocity of atoms with $E = \eta kT$ is $\sqrt{\frac{2\eta kT}{m}} = \sqrt{\pi\eta}\; \bar{v}/2$, where $\bar{v}$ is average thermal velocity; and, fraction of atoms with $\epsilon > \eta kT$ for large η $= 2e^{-\eta}\sqrt{\eta/\pi}$ using a Maxwell-Boltzmann distribution

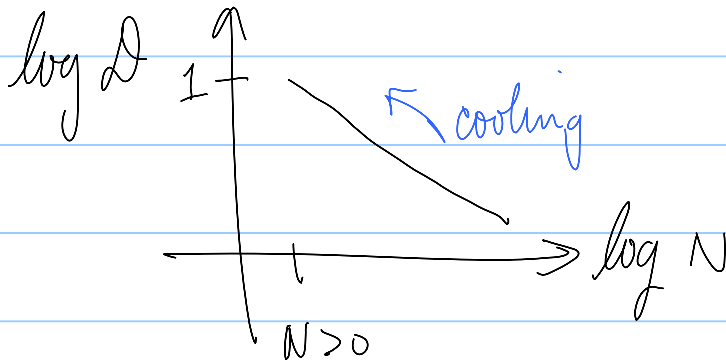
define $\lambda \equiv \frac{\tau_{ev}}{\tau_{el}}$

in the limit of large η, $\lambda \to \frac{\sqrt{2}\, e^{\eta}}{\eta}$

- typically, $\eta \simeq 5$, so $\lambda \simeq 50$

An important fact

$$\gamma = -\frac{d(\ln \mathcal{D})}{d(\ln N)} = \frac{3\alpha}{1+\lambda/R} - 1 = \frac{\eta - 3}{1+\lambda/R} - 1$$



This is what we want

We therefore require $\gamma > 0$ always, which depends on what R is doing for fixed η (η is actually optimized during cooling).

So, we generally need $R > \lambda$ for cooling to degeneracy to work.

In a typical successful cooling trajectory, $R \geq 100$. Keeping $\eta \sim 5$ makes the evaporation speedy and causes τ_{el} to stay the same or get smaller (which is good).

It turns out that: $-\frac{d(\ln R)}{d(\ln N)} = \frac{\alpha}{1+\lambda/R} - 1$

We need R to stay fixed or grow during evaporation; if it starts decreasing, then it will continue to decrease, leading to γ' getting worse and worse.

R increasing is called "runaway evaporation", which is helpful but not necessary to reach $\mathcal{D} \sim 1$.